Carrier concentration in semiconductors

There are two types of carriers:

1- Electrons in the conduction band
2- Holes in the valence band

The electron concentration in the conduction band (CB) is related to the density of available states, $g_c(E)$, and the probability of occupation, $f(E)$. This is given by:

$$n_o = \int_{E_c}^{top} n(E)dE = \int_{E_c}^{top} g_c(E)\, f(E)\, dE \qquad \text{..........(1)}$$

The density of states of the 3D structure is

$$g_c(E) = \frac{8\pi\sqrt{2}}{h^3} m^{*3/2} \sqrt{E - E_c} \qquad \text{........................ (2)}$$

Fermi-Dirac function is

$$f(E) = \frac{1}{1 + exp\left[\frac{E - E_F}{\mathrm{k}T}\right]}$$

For electrons in CB,

$$E > E_F \rightarrow (E - E_F) \gg \mathrm{k}T$$

So,

$$f(E) = \frac{1}{1+exp\left[\frac{E-E_F}{\mathrm{k}T}\right]} \approx exp\left(\frac{-(E-E_F)}{\mathrm{k}T}\right) \qquad \text{....................(3)}$$

Substitute Eq. (2) and (3) in Eq. (1), we get:

$$n_o = \int_{E_c}^{top} \frac{8\pi\sqrt{2}}{h^3} m^{*3/2} \sqrt{E - E_c}\, .\, exp\left(\frac{E_F - E}{\mathrm{k}T}\right) dE$$

$$= \frac{8\pi\sqrt{2}}{h^3} m^{*3/2} .\, exp\left(\frac{E_F}{\mathrm{k}T}\right) \int_{E_c}^{\infty} (E - E_c)^{1/2}\, .\, exp\left(\frac{-E}{\mathrm{k}T}\right) dE$$

Let $E - E_c = x \ \rightarrow dE = dx$

As $E \rightarrow E_c, x = 0$

$E \rightarrow \infty, x = \infty$

So,

$$n_o = \frac{8\pi\sqrt{2}}{h^3} m^{*3/2}. exp\left(\frac{E_F}{kT}\right) \int_0^\infty x^{1/2} \, . exp\left(\frac{-(E_c + x)}{kT}\right) dx$$

$$n_o = \frac{8\pi\sqrt{2}}{h^3} m^{*3/2}. exp\left(\frac{E_F}{kT}\right) exp\left(\frac{-E_c}{kT}\right) \int_0^\infty x^{1/2} \, . exp\left(\frac{-x}{kT}\right) dx$$

$$= \frac{8\pi\sqrt{2}}{h^3} m^{*3/2}. exp\left(\frac{-(E_c - E_F)}{kT}\right) \int_0^\infty x^{1/2} \, . exp\left(\frac{-x}{kT}\right) dx$$

$$= \frac{8\pi\sqrt{2}}{h^3} m^{*3/2}. exp\left(\frac{-(E_c - E_F)}{kT}\right) (kT)^{3/2}. \frac{\sqrt{\pi}}{2}$$

$$\boxed{\begin{gathered} n_o = 2\left(\frac{2\,\pi\, m_n^*\ kT}{h^2}\right)^{3/2} . exp\left(\frac{-(E_c - E_F)}{kT}\right) \\ n_o = N_c. exp\left(\frac{-(E_c - E_F)}{kT}\right) \\ N_c = 2\left(\frac{2\,\pi\, m_n^* k\, T}{h^2}\right)^{3/2} \end{gathered}} \quad \text{......(4)}$$

where $m_n^* = m^*$ which is the effective mass of electrons in CB. N_c is a temperature-dependent constant called the effective density of states at the conduction band edge. It gives the total number of available states per unit volume at the bottom of the conduction band for electrons to occupy. E_c is the bottom of the conduction band and E_F is the position of the Fermi level.

A similar equation can be written for holes in the valence band (VB):

$$\boxed{\begin{array}{c} p_o = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}.\,exp\left(\frac{-(E_F - E_v)}{kT}\right) \\ p_o = N_v.\,exp\left(\frac{-(E_F - E_v)}{kT}\right) \\ N_v = 2\left(\frac{2\,\pi\,m_p^*\,k\,T}{h^2}\right)^{3/2} \end{array}} \qquad ......(5)$$

where $m_p^* = m^*$ which is the effective mass of the hole in VB. N_v is the effective density of states at the valence band edge.

In an **ideally intrinsic semiconductor,** the Fermi level is in the middle of the bandgap and n_o=p_o , so the **intrinsic carrier concentration** can be expressed as

$$n_i^2 = n_o p_o = N_c N_v.\,exp\left(\frac{-(E_c - E_v)}{kT}\right)$$

$$n_i^2 = n_o p_o = N_c N_v.\,exp\left(\frac{-E_g}{kT}\right) \text{......... (6)}$$

$$n_i = \sqrt{N_c N_v}.\,exp\left(\frac{-E_g}{2kT}\right) \qquad \text{......... (7)}$$

$$n_i^2 = n_o p_o \text{ ....(8)}$$

The equation (8) is called **the law of mass action** and it valid for any semiconductor at equilibrium.

Example 1: Find the electron concentration in CB at 300K if N_c (300 K) = 2.8*10^{19} cm^{-3} and E_F is 0.25 eV below E_c.

Solution:

$$n_o = N_c.exp\left(\frac{-(E_C - E_F)}{kT}\right)$$

$$n_o = 2.8 * 10^{19}\,.exp\left(\frac{-0.25}{0.0259}\right) = 1.8 * 10^{15} cm^{-3}$$

Example 2: Find the hole concentration in VB at 400K. N_v(300K) = 1.04*10^{19} cm^{-3} and the difference between the E_F and E_v is 0.27 eV.

Solution:

$$\frac{N_v(400K)}{N_v(300K)} = \left(\frac{400}{300}\right)^{3/2}$$

$$N_v(400K) = 1.04 * 10^{19} * \left(\frac{400}{300}\right)^{3/2} = 1.60 * 10^{19}\ cm^{-3}$$

$$p_o = N_v.exp\left(\frac{-(E_F - E_v)}{kT}\right)$$

$$p_o = 1.60 * 10^{19}\,.exp\left(\frac{-0.27}{0.0345}\right) = 6.43 * 10^{15} cm^{-3}$$

H.W: Find the intrinsic carrier concentration at 300K and 400K for GaAs.

N_c (300 K) = 4.7*10^{17} cm^{-3}

N_v (300K) = 7.0*10^{18} cm^{-3}

E_g = 1.42 eV